Surface plasmons in metallic films of arbitrary thickness with mirror boundary conditions

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Abstract

A metallic film of arbitrary thickness is considered. We show that the problem of description of surface plasma oscillations (surface plasmons) with reflection boundary conditions allows analytic solution. Besides, this problem allows generalization for more general case of conducting matter (in particular, and for semiconductors).

Key words: degenerate plasma, metallic films, dielectric permeability, metal films, reflection boundary conditions.

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Introduction

Problem about surface plasma oscillations long time draws to itself attention [1]–[5]. It is connected as with theoretical interest to this problem, and with numerous practical appendices. Thus the majority of researches is devoted research surface plasma waves on border of two substances. At the same time The big interest causes process of distribution of the surface plasma waves in thin films, in particular, the metal.

Researches of interaction of an electromagnetic wave with metal film were spent basically for a case mirror dispersion of electrons on a film surface. The macroscopical approach was thus used. The kinetic equations thus were not used. In the present work it is shown, that for films any thickness the problem with application of the kinetic approach supposes the analytical solution for mirror boundary conditions.

For more general boundary conditions a problem for a film of any thickness essentially becomes complicated and does not suppose generally the analytical solution.

Let's notice, that the most part of our reasonings will be fair for more the general case conducting medium (in particular and semiconductor) films.

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1. The basic equations

Let's consider a metal film of any thickness. We take the Cartesian system of coordinates with the origin of coordinates with an axis x, directed perpendicularly to the surface of a film. Axis z we will direct along a direction of distribution of the surface electromagnetic wave. We will notice, that in this case a magnetic field it is directed along an axis y. The origin of coordinates we will place in the middle of a film. Let's designate a thickness film through d.

Out of a film the electromagnetic field is described by the wave equations

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \triangle \mathbf{E} = 0,$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} - \triangle \mathbf{H} = 0.$$

Here $\mathbf{E} = \{E_x, 0, E_z\}$, $\mathbf{H} = \{0, H_y, 0\}$ are vectors of electric and magnetic fields strength, c is the speed of light.

The solution of these equations decreasing on infinity, looks like

$$\mathbf{E} = \begin{cases} \mathbf{E}_1 e^{-i\omega t + \alpha x + ikz}, & x < -d/2, \\ \mathbf{E}_2 e^{-i\omega t - \alpha x + ikz}, & x > d/2, \end{cases} \tag{1a}$$

$$\mathbf{H} = \begin{cases} \mathbf{H}_1 e^{-i\omega t + \alpha x + ikz}, & x < -d/2, \\ \mathbf{H}_2 e^{-i\omega t - \alpha x + ikz}, & x > d/2. \end{cases} \tag{1b}$$

In the equations (1a) and (1b) ω is the frequency of a wave, k is the wave number. The attenuation parameter α is connected with these quantity by equation

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{c^2}}. \tag{2}$$

Then behaviour of electric and magnetic fields of a wave in a film it is described by the following system of differential equations [4]:

$$\begin{cases} \dfrac{dE_z}{dx} - ikE_x + \dfrac{i\omega}{c}H_y = 0, \\[2mm] \dfrac{i\omega}{c}E_x - ikH_y = \dfrac{4\pi}{c}j_x, \\[2mm] \dfrac{dH_y}{dx} + \dfrac{i\omega}{c}E_z = \dfrac{4\pi}{c}j_z. \end{cases} \tag{3}$$

Here $\mathbf{j}$ is the current density.

Let's consider two cases of a configuration of external fields. In the first case we will consider a symmetric configuration of y–components of a magnetic field and x–components electric fields, and an antisymmetric configuration of z–components electric field, i.e.

$$H_y\left(-\frac{d}{2}\right) = H_y\left(\frac{d}{2}\right), \quad E_x\left(-\frac{d}{2}\right) = E_x\left(\frac{d}{2}\right), \quad E_z\left(-\frac{d}{2}\right) = -E_z\left(\frac{d}{2}\right).$$

In the second case we will consider an antisymmetric configuration of y–components of a magnetic field and x–components electric fields, and a symmetric configuration of z–components electric field, i.e.

$$H_y\left(-\frac{d}{2}\right) = -H_y\left(\frac{d}{2}\right), \quad E_x\left(-\frac{d}{2}\right) = -E_x\left(\frac{d}{2}\right), \quad E_z\left(-\frac{d}{2}\right) = E_z\left(\frac{d}{2}\right).$$

2. Dielectric permeability and impedance

The impedance in both cases is thus certain equally as follows

$$Z_j = \frac{E_z(-0)}{H_y(-0)}, \qquad j = 1, 2.$$

Let's consider the case of mirror reflection of electrons from a film surface. Then for quantities $Z_j\,(j = 1, 2)$ we have the following equalities [1]

$$Z_j = -\frac{2i}{W(d)\Omega} \sum_{n=-\infty}^{n=\infty} \frac{1}{Q^2} \left[\frac{Q_x^2}{\varepsilon_{tr} - Q^2/\Omega^2} + \frac{Q_z^2}{\varepsilon_l} \right], \quad j = 1, 2. \qquad (4)$$

And for $Z^{(1)}$ summation is conducted on odd n, and for $Z^{(2)}$ on even, ε_{tr} and ε_l are accordingly the transvercal and longitudinal dielectric permeability of plasma, $\Omega = \omega/\omega_p$, $\varepsilon = \nu/\omega_p$, ω_p is the plasma (Langmuir) frequency of plasma oscillations, $\omega_p^2 = 4\pi e^2 N/m$, e and m are charge and mass of electron, N is the concentration (numerical density) of electrons, $W(d) = \omega_p d/c$, the wave vector looks like

$$\mathbf{Q} = \frac{c}{\omega_p}\mathbf{q}, \qquad \mathbf{q} = \left\{\frac{\pi n}{d}, 0, k\right\},$$

$$q = |\mathbf{q}| = \sqrt{\frac{\pi^2 n^2}{d^2} + k^2}, \qquad n = 0, \pm 1, \pm 2, \cdots,$$

k is the dimensional wave number.

Let's introduce an unit vector

$$\mathbf{e} = \frac{\mathbf{q}}{q} = \left\{\frac{Q_x}{Q}, 0, \frac{Q_z}{Q}\right\} = \frac{1}{q}\left\{\frac{\pi n}{d}, 0, k\right\}.$$

Then the formula (4) will be copied in the form

$$Z_j = -\frac{2i}{W(d)\Omega} \sum_{n=-\infty}^{n=\infty} \left[\frac{e_x^2}{\varepsilon_{tr} - Q^2/\Omega^2} + \frac{e_z^2}{\varepsilon_l} \right], \quad j = 1, 2,$$

where $e_x = \dfrac{\pi n}{qd}$, $e_z = \dfrac{k}{q}$, or, in explicit form

$$Z_j = -\frac{2i}{W(d)\Omega} \sum_{n=-\infty}^{n=\infty} \frac{1}{q^2} \left[\frac{(\pi n/d)^2}{\varepsilon_{tr} - (cq/\Omega\omega_p)^2} + \frac{k^2}{\varepsilon_l} \right], \quad j = 1, 2, \qquad (5)$$

where

$$W(d) = \frac{\omega_p}{c}d.$$

Further we introduce the quantity

$$q_1 = \frac{v_F}{\omega_p}q = \frac{v_F}{\omega_p}\sqrt{\frac{\pi^2 n^2}{d^2} + k^2}.$$

By means of this quantity the transvercal conductivity is calculated by the formula

$$\varepsilon_{tr} = 1 - \frac{3}{4\Omega q_1^3} \left[2(\Omega + i\varepsilon)q_1 + \left[(\Omega + i\varepsilon)^2 - q_1^2 \right] \ln \frac{\Omega + i\varepsilon - q_1}{\Omega + i\varepsilon + q_1} \right],$$

or

$$\varepsilon_{tr} = 1 - \frac{3}{4\Omega q_1} \left\{ 2\frac{\Omega + i\varepsilon}{q_1} + \left[\left(\frac{\Omega + i\varepsilon}{q_1} \right)^2 - 1 \right] \ln \frac{(\Omega + i\varepsilon)/q_1 - 1}{(\Omega + i\varepsilon)/q_1 + 1} \right\}.$$

Longitudinal dielectric permeability of degenerate plasmas is equal to:

$$\varepsilon_l = 1 + \frac{3}{q_1^2} \frac{1 + \dfrac{\Omega + i\varepsilon}{2q_1} \ln \dfrac{\Omega + i\varepsilon - q_1}{\Omega + i\varepsilon + q_1}}{1 + \dfrac{i\varepsilon}{2q_1} \ln \dfrac{\Omega + i\varepsilon - q_1}{\Omega + i\varepsilon + q_1}}.$$

If to introduce the variable

$$Z = \frac{\Omega + i\varepsilon}{q_1},$$

that expressions for transversal and longitudinal permeability will write down more shortly:

$$\varepsilon_{tr} = 1 - \frac{3}{4\Omega q_1} \left[2Z + (Z^2 - 1) \ln \frac{Z - 1}{Z + 1} \right],$$

$$\varepsilon_l = 1 + \frac{3}{q_1^2}\left[1 + \frac{Z}{2}\ln\frac{Z-1}{Z+1}\right]\left[1 + \frac{i\varepsilon}{2q_1}\ln\frac{Z-1}{Z+1}\right]^{-1}.$$

Let's transform now functions Z_1 and Z_2. For the first function according to (5) it is received:

$$Z_1 = -\frac{4i}{\omega d^3}\sum_{n=1}^{+\infty}\frac{1}{q_{2n-1}^2}\left[\frac{\pi^2(2n-1)^2}{\varepsilon_{tr}(q_{1,2n-1}) - c^2 q_{2n-1}^2/\omega^2} + \frac{(kd)^2}{\varepsilon_l(q_{1,2n+1})}\right],$$

where

$$q_{1,2n+1} = \frac{v_F}{\omega_p}q_{2n-1}, \qquad q_{2n-1} = \sqrt{\frac{\pi^2}{d^2}(2n+1)^2 + k^2}.$$

For the second function according to (5) it is received following expression

$$Z_2 = -\frac{2ick^2}{\omega d\varepsilon_l(q_{1,0})} - \frac{4ic}{\omega d^3}\sum_{n=1}^{\infty}\frac{1}{q_{2n}^2}\left[\frac{\pi^2(2n)^2}{\varepsilon_{tr}(q_{1,2n}) - c^2 q_{2n}^2/\omega^2} + \frac{(kd)^2}{\varepsilon_l(q_{2n})}\right],$$

where

$$q_{2n} = \sqrt{\frac{\pi^2}{d^2}(2n)^2 + k^2}, \qquad q_{1,2n} = \frac{v_F}{\omega_p}q_{2n}.$$

3. Surface plasmon in case of symmetry on a magnetic field

From the third equation of system (3) it is received the following communication between projections electric and magnetic fields nearby from the bottom surface of a layer out of it (when $j_z = 0$)

$$\alpha H_y(0) = -\frac{i\omega}{c}E_z(0),$$

whence for the impedance we obtain the expression

$$Z_j = \frac{i\alpha c}{\omega}, \qquad j = 1, 2. \tag{6}$$

Considering the equation (2) expression (6) can be transformed to the form

$$\sqrt{k^2 - \frac{\omega^2}{c^2}} = -\frac{i\omega}{c}Z_j, \qquad j = 1, 2. \tag{7}$$

For thin films and small k earlier we had obtained the following expression [11] $\omega^2 = 4(ck)^2/(4 + k^2 d^2)$.

Within the limits of macroscopical electrodynamics for great values of wave number k for frequency of surface plasma oscillations fairly following equation [10]

$$\omega^{\pm}(k) = \sqrt{\frac{1 \pm e^{-kd}}{2}}\,\omega_p.$$

Let's consider the case when the film consists of a layer of potassium. Then [1]$\omega_p = 6.5 \times 10^{15} sec^{-1}$, $v_F = 8.52 \times 10^7$ cm/sec.

On fig. 1–6 we will present dependence of the real and imaginary parts of frequency plasma oscillations from wave number for films with the thickness equals to 5, 10 and 50 nanometers.

From drawings 1–3 it is visible, that for thin films (in particular, with the thickness equals to 5 and 10 nm) the real part of frequency of plasma oscillations has the maximum near to wave number $k = 1$. Thus at $0 < k < 1$ there is a sharp increase of the real part of frequency of oscillations, at $k > 1$ goes its smooth decrease. With growth of a thickness of a film the maximum of the real part frequencies of plasma oscillations is erased. The quantity Re $\Omega(k)$ becomes monotonously increasing.

On fig. 4–6 dependence of an imaginary part of frequency of plasma oscillations from wave number is presented. Irrespective of a thickness films this dependence is monotonously decreasing. Near to the wave numbers $k = 1$, to be exact, at $0 < k < 1$ occurs sharp decrease of the imaginary parts of frequency of plasma oscillations, and at $k > 1$ such decrease is smooth. With growth of a thickness of a film sharp decrease near to a point $k = 1$ smoothes out.
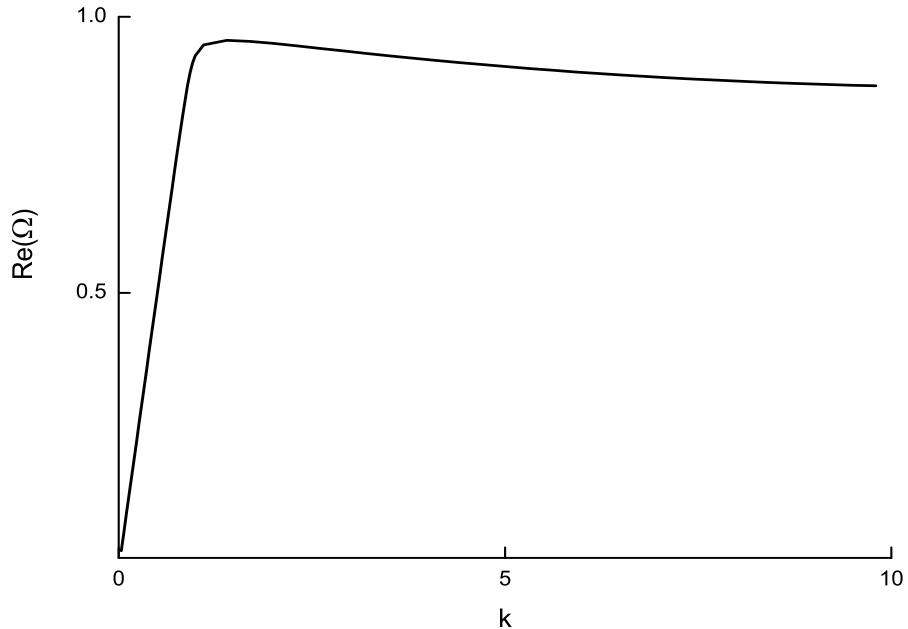

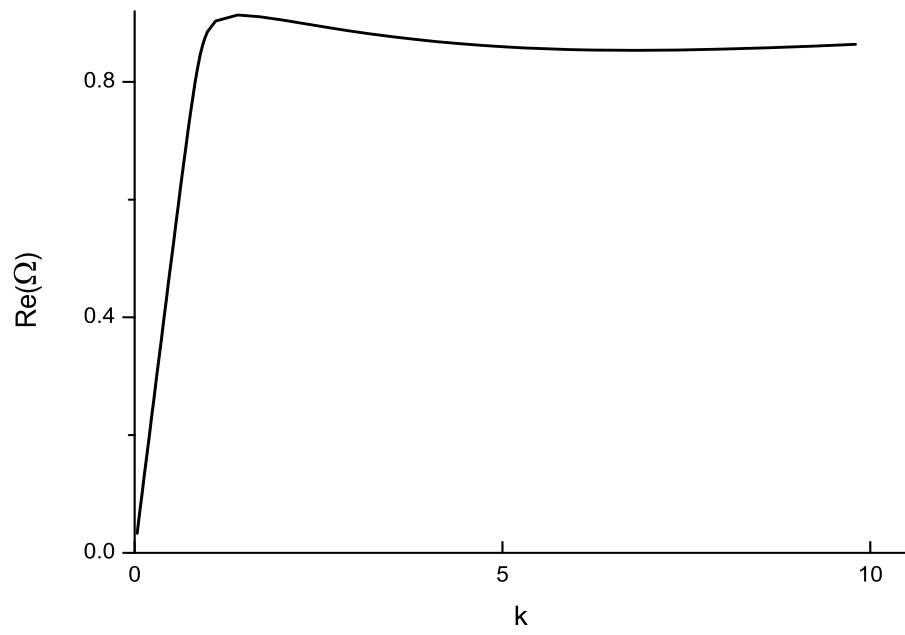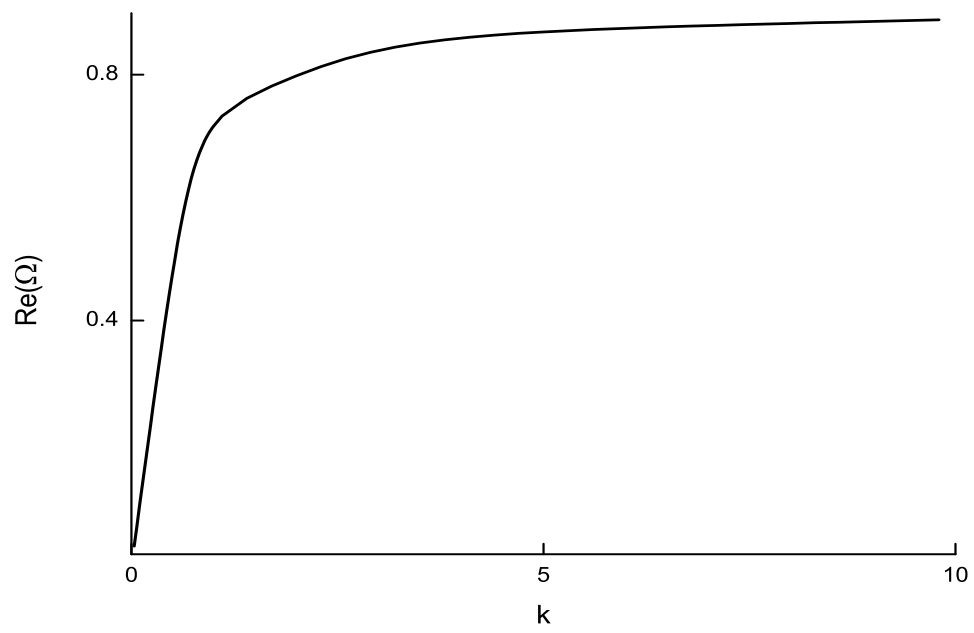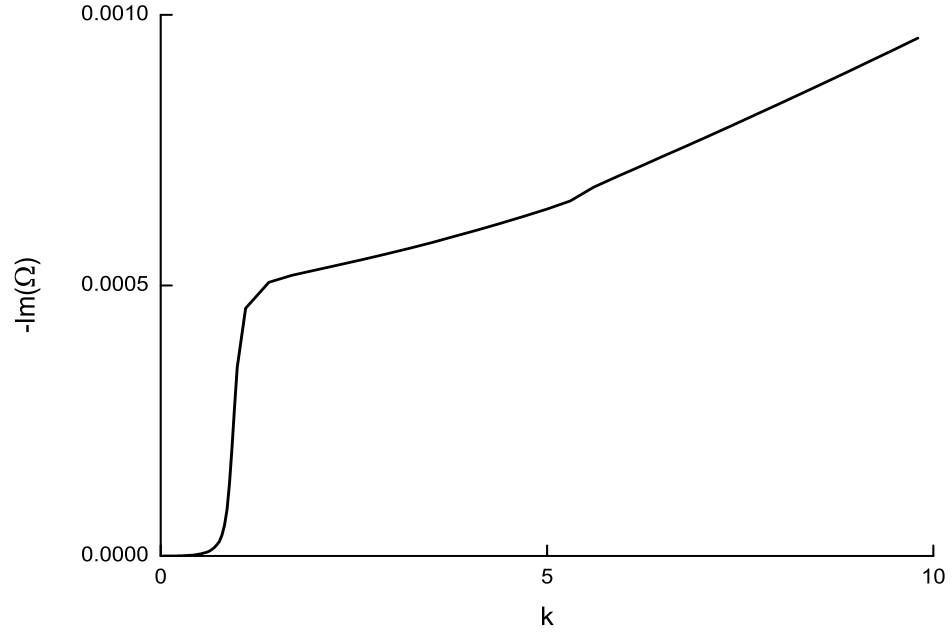Figure 1: Surface plasmon. The thickness of film equals 5 nm. The dependence Re $\Omega(k)$.
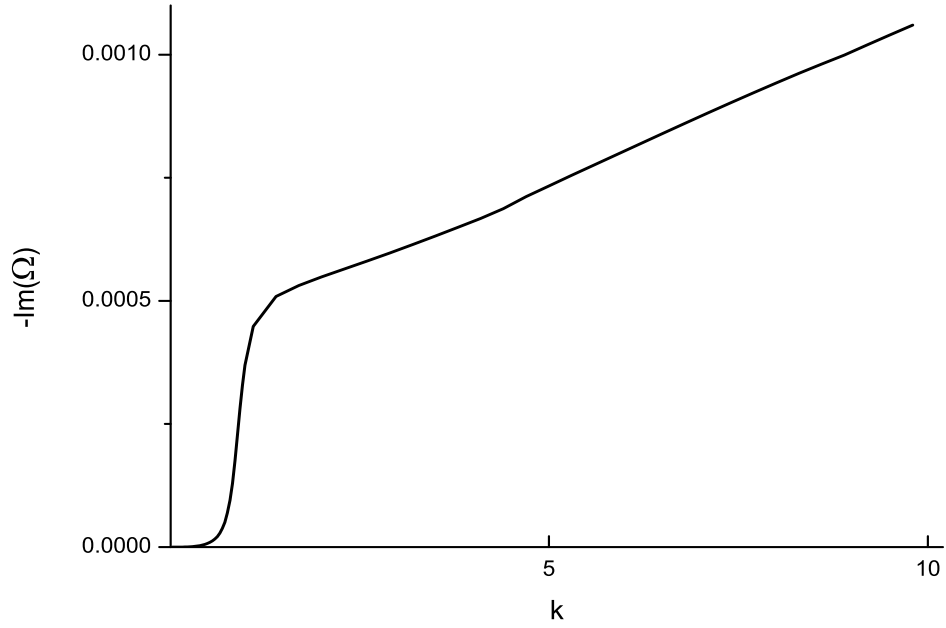
Figure 2: Surface plasmon. The thickness of film equals 10 nm. The dependence $\mathrm{Re}\,\Omega(k)$.

Figure 3: Surface plasmon. The thickness of film equals 50 nm. The dependence $\mathrm{Re}\,\Omega(k)$.

Figure 4: Surface plasmon. The thickness of film equals 5 nm. The dependence $-\operatorname{Im} \Omega(k)$.

Figure 5: Surface plasmon. The thickness of film equals 10 nm. The dependence $-\operatorname{Im} \Omega(k)$.
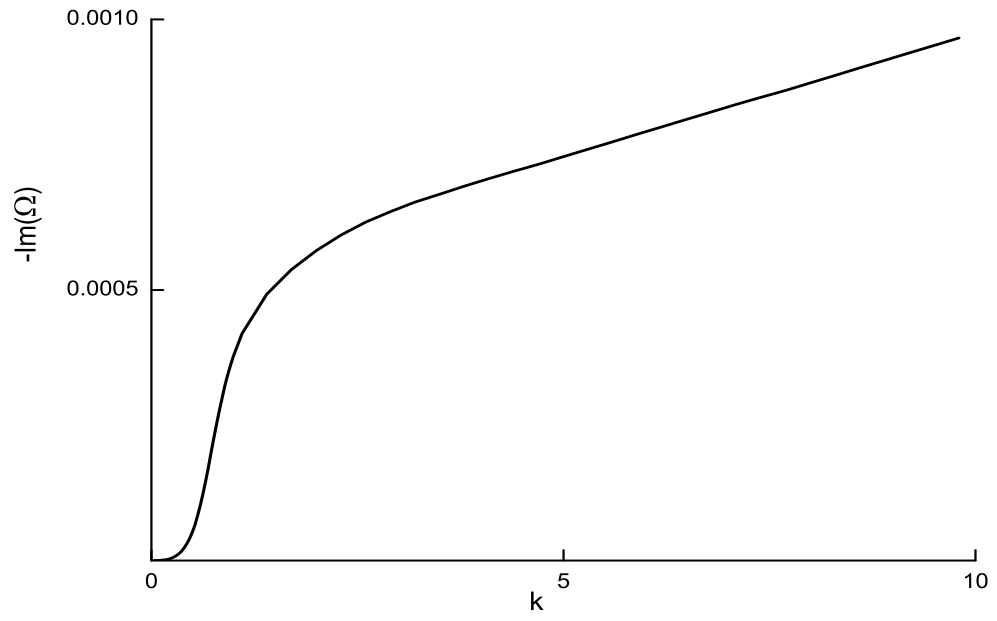
Figure 6: Surface plasmon. The thickness of film equals 50 nm. The dependence $-\operatorname{Im}\Omega(k)$.

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